

Field tuned atom-atom entanglement via dipole-dipole interaction

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Abstract

We propose a simple scheme, in which only one atom couples to a cavity field, to entangle two two-level atoms. We connect two atoms with dipole-dipole interaction since one of them can move around the cavity. The results show that the peak entanglement does not depend on dipole-dipole interaction strength but on field density at a certain controlling time. So the field density can act as a switch for maximum entanglement (ME) generation.

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1 Introduction

Recently, generation of entanglement in atomic systems has been intensely paid attention to because of the motivations in the potential applications in quantum information[1, 2, 3] and computation processing[4, 5]. The realization of easily controllable atomic entangled states turns out to be one of the crucial and challenging tasks. A number of schemes of generating entanglement between atoms have been put forward to realizing quantum teleportation[6, 7, 8] and swapping[9, 10]. In many of the cases, atoms are trapped in cavities or potential wells so that they can be connected through directly exchanging real photons and then they can be strongly entangled. While, practical applications in quantum information processing require *engineering* entangled atoms[11]. This expects operable atoms so that they can be moved to distance without losing of information. To overcome this obstacle, many schemes have been

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proposed[12, 13, 14, 15], for example, in Ref. [12], the atoms are assumed to move in and out of the optical cavities and precisely received by detectors, maximum entanglement (ME) are acquired by fixing controlling time. In this paper, we propose a simple scheme to realize an easily engineered two-atom entangled state. The advantage of this scheme is only one atom is trapped in a cavity, and the other one can be spatially moved freely outside the cavity. Further more, the amount of the entanglement can be manually tuned by adjusting some key observable variables of the system.

2 Model Description

We consider a system constituted by two two-level identical atoms (1, 2) and a single mode electromagnetic cavity field, as is shown in Fig. 1. Atom 1 resonantly interacting with the cavity field (sinelike line in the diagram) is trapped in a slender microcavity. Atom 2 lies beside atom 1 out of the cavity. We assume the cavity is so much narrow that atom 2 is close to atom 1. If the relative distance of two atoms and the de Broglie wavelength of two atoms can compare, the dipole-dipole interaction (dashed line connecting two atoms in the diagram) should be included. The whole device is located in vacuum.

The Hamiltonian of this system can be written as

$$H = \hbar\omega(\sigma_1^z + \sigma_2^z) + \hbar\omega a^\dagger a + g(a\sigma_1^+ + h.c.) + \Gamma(\sigma_1^+ \sigma_2^- + h.c). \quad (1)$$

where $\sigma_1^z, \sigma_2^z, \sigma_1^\pm$ and σ_2^\pm are spin operators and raising (lowering) operators of atom 1 and 2 respectively, $a^\dagger, (a)$ is the creation (annihilation) operator of field, g is the coupling strength of atom 1 to field, while Γ is the coefficient of atom-atom dipole-dipole interaction which has been considered by many authors[16, 17]. In the invariant sub-space of the global system, we can choose a set of complete basis of atom-field system as $|g, g, n+2\rangle, |e, g, n+1\rangle, |g, e, n+1\rangle, |e, e, n\rangle$. On this basis, the Hamiltonian of the system can be written as

$$H = \begin{pmatrix} (n+1)\omega & g\sqrt{n+2} & 0 & 0 \\ g\sqrt{n+2} & (n+1)\omega & \Gamma & 0 \\ 0 & \Gamma & (n+1)\omega & g\sqrt{n+1} \\ 0 & 0 & g\sqrt{n+1} & (n+1)\omega \end{pmatrix}. \quad (2)$$

We can easily get four eigenvalues of this Hermite Hamiltonian as

$$E_{12} = (n+1)\omega \pm A, E_{34} = (n+1)\omega \pm B, \quad (3)$$

where, $A = \sqrt{C+D}/2$, $B = \sqrt{C-D}/2$ with $C = 4ng^2 + 6g^2 + 2\Gamma^2$, $D = 2\sqrt{4(n+1)g^2\Gamma^2 + (g^2 + \Gamma^2)^2}$. Also, the corresponding four eigenvectors are

$$|\phi_i\rangle = \xi_i [x_{i1} |g, g, n+2\rangle + x_{i2} |e, g, n+1\rangle + x_{i3} |g, e, n+1\rangle + x_{i4} |e, e, n\rangle] \quad (4)$$

with $i = 1, 2, 3, 4$, $\xi_i = 1/\sqrt{|x_{i1}|^2 + |x_{i2}|^2 + |x_{i3}|^2 + |x_{i4}|^2}$, and x_{ij} ($j = 1, 2, 3, 4$) satisfy $x_{i1} = 1$, $x_{i2} = (-1)^{i+1} \frac{C}{(\sqrt{n+2}g)}$, $x_{i3} = \frac{[C^2 - (n+2)g^2]}{\sqrt{n+2}g\Gamma}$, $x_{i4} = (-1)^{i+1} \frac{C[C^2 - (n+2)g^2 - \Gamma^2]}{\sqrt{n+1}\sqrt{n+2}g^2\Gamma}$, where $C = A$ for odd i and $C = B$ for even i .

For a given initial state $\psi(0)$ of system, we can obtain the evolved dressed state of system $\psi(t)$ which is controlled by an unitary transformation $U = e^{-iHt/\hbar}$. Also, $\psi(t)$ can be expanded as a superposition of eigenstates ϕ_i

$$|\psi(t)\rangle = \sum_i C_i(t) |\phi_i\rangle. \quad (5)$$

The coefficients $C_i(t)$ are determined by solving Schrodinger Equation, so that

$$C_i(t) = C_i(0)e^{-iE_it/\hbar}. \quad (6)$$

The reduced density matrix of two-atom is obtained by tracing over the field variables of system density matrix $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$, that is $\rho_{atom}(t) = Tr_f \rho(t) = \sum_n \langle n | \rho(t) | n \rangle$. The initial condition $C_i(0)$ is easily given for an initial system state $|\psi(0)\rangle = |\psi(0)\rangle_{atom} \otimes |\psi(0)\rangle_{field}$.

In the next section, we will discuss the two-atom entanglement induced by the dipole-dipole interaction between atoms.

3 Two-atom Entanglement Nature Under Cavity Field

Wootters Concurrence, which has been proved to be effective in measuring the entanglement of two qubits, is defined as[18]

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (7)$$

where λ_i are four non-negative square roots of the eigenvalues of the non-hermitian matrix $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order. In dealing with this model, the Concurrence is simply determined by several density matrix elements since

most of the off-diagonal elements are eliminated due to the adiabatic evolution. We choose the initial system state is a separable pure state as

$$|\psi(0)\rangle = |g\rangle_2 \otimes |g\rangle_1 \otimes |n_0\rangle. \quad (8)$$

Then the evolution of system state is determined by a series of parameters space (n, g, Γ) . Fig. 2 shows two-atom entanglement under parameters space $(1, 5.0, 0.5)$ for solid line and $(1, 5.0, 0.1)$ for dashed line.

Generally, the entanglement shows local peaks, all the peaks present to be covered under series of wave packets. This is caused by the Rabi oscillation of atoms, and in fact the synchronization difference between the coupling of atom 1 to field and atom 1 to atom 2. The curves show that *larger* dipole-dipole interaction can improve the local peak of the entanglement but take no effect on the width of the peak of the amount of entanglement. Physically, dipole-dipole interaction Γ can be enhanced by reducing the relative distance between two atoms or increasing the dipole polar moment for each atom[19].

Fig. 3 shows the influence of different coupling of atom 1 to cavity on the two-atom entanglement under same dipole-dipole coupling strength. Surprisingly, weak atom-field coupling amplifies the packets of the entanglement so that the amount and the width of each peak of the entanglement are both enlarged. Especially, the first packet of the entanglement is amplified most evidently. Since the coupling of intro-cavity atom and cavity depends on the relative position of atom, $\mathbf{r}(x, y, z)$, in the cavity, such that $g = g_0 \sin(k_0 z) \exp[-(x^2 + y^2)/\omega_0^2]$ with g_0 the peak coupling rate, k_0 and ω_0 the wave vector and width of the cavity mode[20]. The ideal case would be fixing the atom to keep g a constant so that the controlling time of peak entanglement can be precisely operated.

In Fig. 4, we depict the entanglement under different intro-cavity field densities n . The alternating of field density does not seem to take influence on the local peak of the amount of the entanglement. While, the width of the packet is slightly broadened for larger n . To distinctly represent the entanglement under this situation, we illustrate two-atom entanglement versus initial field density and dipole-dipole interaction strength in Fig. 5. In computing the results, the controlling time is restricted to be 4. Note that the ME can never be reached when Γ is small for arbitrary n . Surprisingly, the local peak entanglement arises for a series of fixed n whatever the value of Γ be, for example, the first peak emerges at about $n = 4$, the second at about $n = 14$. This suggests us the initial field density acts as a tuning switch that discretely controls the generation

of ME at any specific time whatever the dipole-dipole interaction strength be. Certainly, appropriate Γ is optimal, since the ME arises at about $\Gamma = 0.4$ when $t = 4$ and $n = 4, 14, \dots$, in this system.

4 Conclusion

We proposed a simple scheme to entangle two two-level atoms that can be realized physically. The Hamiltonian for the system was diagonalized to obtain the eigenvalues. For a given initial global state, the evolutive state of the two-atom subsystem was found to be entangled. The amount of the entanglement is presented to depend on the coupling strength g of intra-cavity atom to field and the atomic dipole-dipole interaction strength Γ . It was found that larger Γ and smaller g benefit the quantity and quality of peak entanglement. When these two characters were fixed, the entanglement would be determined by the cavity field density and the controlling time. And for a fixed controlling time, the ME took place at a series of discrete field density intervals. From this point of view, the field density n acts as a ME generation tuning switch. The advantage of this scheme is the extra-cavity atom can be moved so that the relative distance between two atoms can be controlled, then Γ is controllable. Also, field density n can be alternated using high quality laser jet. So, this device may be further developed into an engineered atom-atom entangler.

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References

- [1] S. B. Zheng and G. C. Guo, Phys. Rev. Lett. 85, 2392(2000).
- [2] L. You, X. X. Yi, and X. H. Su, Phys. Rev. A 67, 032308(2003).
- [3] X. X. Yi, X. H. Su, and L. You, Phys. Rev. Lett. 90, 097902(2003).
- [4] A. Beige, Phys. Rev. A 67, 020301(2003).
- [5] K. G. H. Vollbrecht, E. Solano, and J. I. Cirac, Phys. Rev. Lett. 93, 220502(2004).
- [6] S. B. Zheng, Phys. Rev. A 69, 064302(2004).

- [7] G. Chimczak, R. Tanaś, and A. Miranowicz, Phys. Rev. A **71**, 032316 (2005).
- [8] J. Cho and H. W. Lee, Phys. Rev. A **70**, 034305(2004).
- [9] A. Kuzmich¹, and E. S. Polzik, Phys. Rev. Lett. **85**, 5639(2000).
- [10] L. You and M. S. Chapman, Phys. Rev. A **62**, 052302(2000).
- [11] A. Widera, O. Mandel, M. Greiner, S. Kreim, T. W. Hänsch, and I. Bloch, Phys. Rev. Lett. **92**, 160406(2004).
- [12] C. Marr, A. Beige, and G. Rempe, Phys. Rev. A **68**, 033817(2003).
- [13] S. Mancini and S. Bose, Phys. Rev. A **70**, 022307(2004).
- [14] G. Chimczak, Phys. Rev. A **71**, 052305(2005).
- [15] J. K. Asbóth, P. Domokos, and H. Ritsch, Phys. Rev. A **70**, 013414(2004).
- [16] T. J. Carroll, K. Claringbould, A. Goodsell, M. J. Lim, and M. W. Noel, Phys. Rev. Lett. **93**, 153001(2004).
- [17] T. Opatrny, B. Deb, and G. Kurizki, Phys. Rev. Lett. **90**, 250404(2003).
- [18] K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
- [19] R. D. Griffin and S. M. Harris, Phys. Rev. A **25**, 1528(1982).
- [20] L. M. Duan, A. Kuzmich, and H. J. Kimble, Phys. Rev. A **67**, 032305(2002).

Figure Captions:

Fig. 1: Schematic diagram for field modulated two-atom entanglement model. One atom is trapped by a electromagnetic field in a microcavity, the other is located not far away beside the former atom out of the cavity. Two atoms can be connected through dipole-dipole interaction.

Fig. 2: The evolution of two-atom entanglement versus time under different dipole-dipole coupling. Solid line for $\Gamma = 0.5$, dashed line for $\Gamma = 0.1$.

Fig. 3: The evolution of two-atom entanglement versus time under different atom-field coupling. Solid line for $g = 1$, dashed line for $g = 0.5$.

Fig. 4: The evolution of two-atom entanglement versus time under different initial field densities. Solid line for $n = 5$, dashed line for $n = 6$.

Fig. 5: The evolution of two-atom entanglement versus initial field density and atomic dipole-dipole interaction strength. The controlling time is fixed at $t = 4$.









